

A decorative vertical strip on the left side of the page, featuring a black background with a white grid of intersecting lines forming a pattern of small squares and triangles.

Mathematics

HP COMPUTER CURRICULUM

Functions

TEACHERS ADVISOR

HEWLETT  PACKARD

Hewlett-Packard
Computer Curriculum Series

mathematics
TEACHER'S ADVISOR

functions

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Mathematics Set of the Hewlett-Packard Computer Curriculum Series consists of a Student Lab Book and a corresponding Teacher's Advisor. It was designed to help meet the need for computer-oriented problems in mathematics, providing students an opportunity to use a computer as a problem solving tool within a particular subject matter area.

The materials are designed for flexible use as desired by the individual instructor. The material and exercises in this unit are intended as an "enrichment" experience in the field of functions. The unit begins with a standard definition of a function. Then, in the first of two major sections, finite functions are used to illustrate such properties as the inverse of a function, increasing and decreasing functions, and symmetry. In the second section, infinite functions are used to investigate other properties such as the zeros of a function, continuity, and relative maximums and minimums. The unit is not intended to be a complete study of functions; instead, each discussion is self-contained, so the concepts can be studied independently and in any sequence. Thus, the material can be used to supplement and enrich your curriculum in any fashion you choose.

The mathematical concepts needed for each exercise are briefly reviewed, but you may want your students to study these in greater detail before attempting the exercises, especially if their background in functions is limited. A list of possible references is included at the end of each of the two major sections. These references will also provide additional problems for your better students.

The degree of difficulty of the material is dependent upon the amount of assistance which you choose to provide. With no assistance, the better student should be challenged. However, given a good deal of assistance, any second year algebra student should be able to work out the exercises with no great difficulty. The level of the material is determined by the assumption that students taking second year algebra will be quite capable as a group.

The Lab Book provides text material and programming exercises for the students. There is a problem analysis, including a suggested approach and a macro flow chart, for each exercise. The Teacher's Advisor contains an example of a program to solve each exercise, micro flow charts, and a brief discussion of the important elements of the exercise. The micro flow charts should be given to the students only after they have made an attempt to solve the problem on their own. For best results, you should study all the solutions until you are certain you have a complete grasp of the general methods. This should be done before assigning the material to the class.

You will undoubtedly think of different programming methods or techniques as you study the example programs. Encourage the students to do the same. There are no *approved* solutions. All solutions are acceptable if they produce the correct results. At this level, there is no need for emphasis on the efficiency of a student's program. The important question is, does it work?

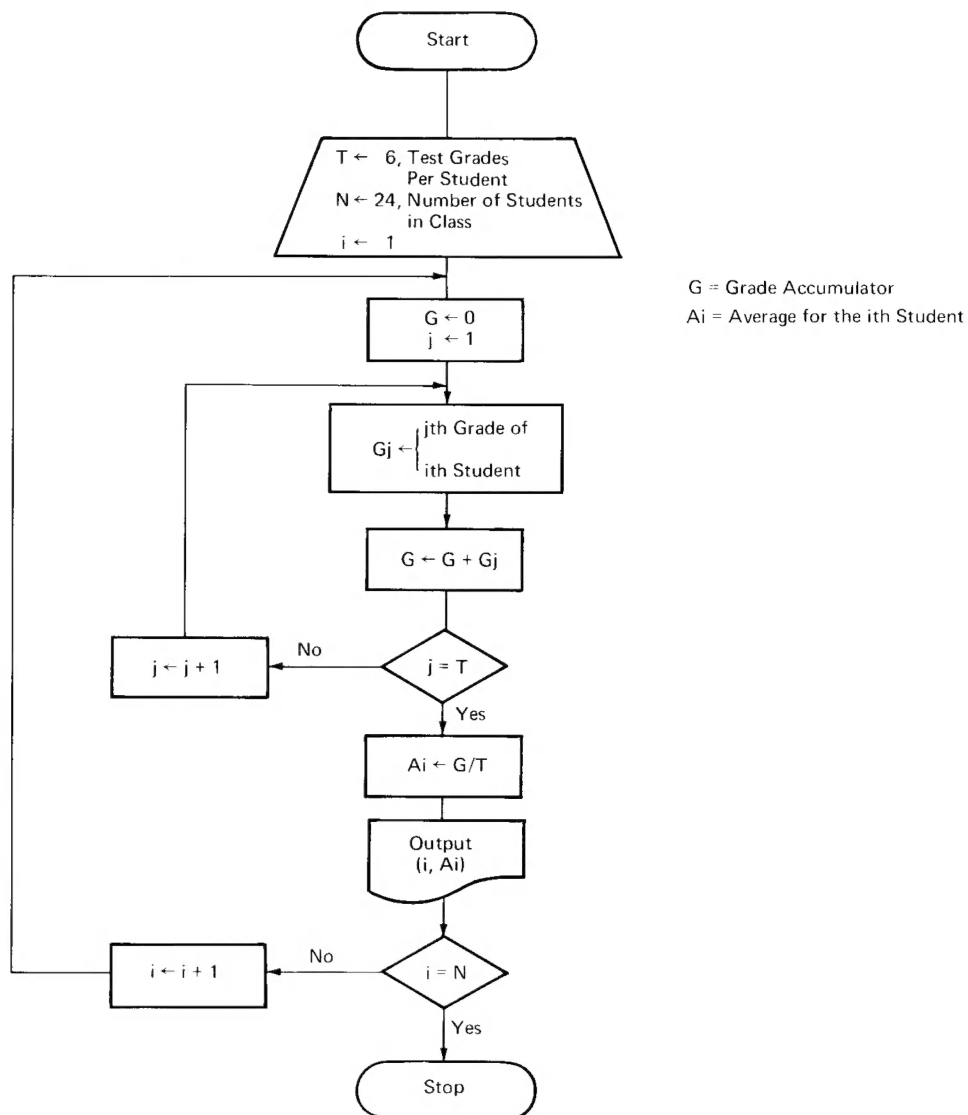
DEFINITION OF A FUNCTION

Exercise 1 — Averaging Grades as an Example of a Function

(a) A program to compute the grade average for a class of 24 students for a 9 week grading period. The output is printed in ordered pair form.

Micro Flow Chart

Exercise 1(a)



Example Program

Exercise 1(a)

```
10 REM--THIS PROGRAM WILL COMPUTE THE AVERAGE GRADE FOR A CLASS
11 REM--OF N STUDENTS BASED ON T GRADES FOR ONE GRADING PERIOD.
12 REM--THE OUTPUT IS IN ORDERED PAIR (FUNCTION) FORM.  ENTER
13 REM--ON THE DATA LINE N,T (NUMBER OF STUDENTS,NUMBER OF TESTS) AND
14 REM--THE TEST SCORES OF EACH STUDENT, NO.1 THROUGH NO. N.
15 DATA 24,6,100,90,98,72,96,88,100,95,90,100,98,100,60,80,62,63,76,68
16 DATA 100,70,84,90,80,80,100,90,92,70,88,78,85,65,84,70,84,74
17 DATA 100,100,94,92,94,100,100,95,90,87,96,100,100,70,73,80,88,80
18 DATA 100,75,82,87,84,88,100,90,88,100,94,96,70,75,81,80,84,80
19 DATA 100,65,88,72,80,80,85,90,88,87,86,84,80,70,46,55,70,64,75,58
20 DATA 57,70,56,80,65,66,66,74,60,100,78,73,75,60,96,95,55,61,60,60
21 DATA 100,90,61,57,58,60,80,85,85,70,80,64,80,80,80,40,68,84
22 DATA 100,50,84,75,84,76,85,50,54,68,50,85,60,56,55,74,68
25 DIM A[200]
30 READ N,T
50 FOR I=1 TO N
55 LET G=0
60 FOR J=1 TO T
70 READ G[J]
80 LET G=G+G[J]
90 NEXT J
100 LET A[I]=G/T
110 PRINT "(STUDENT NO. ";I;" , "A[I];")"
120 NEXT I
130 END
```

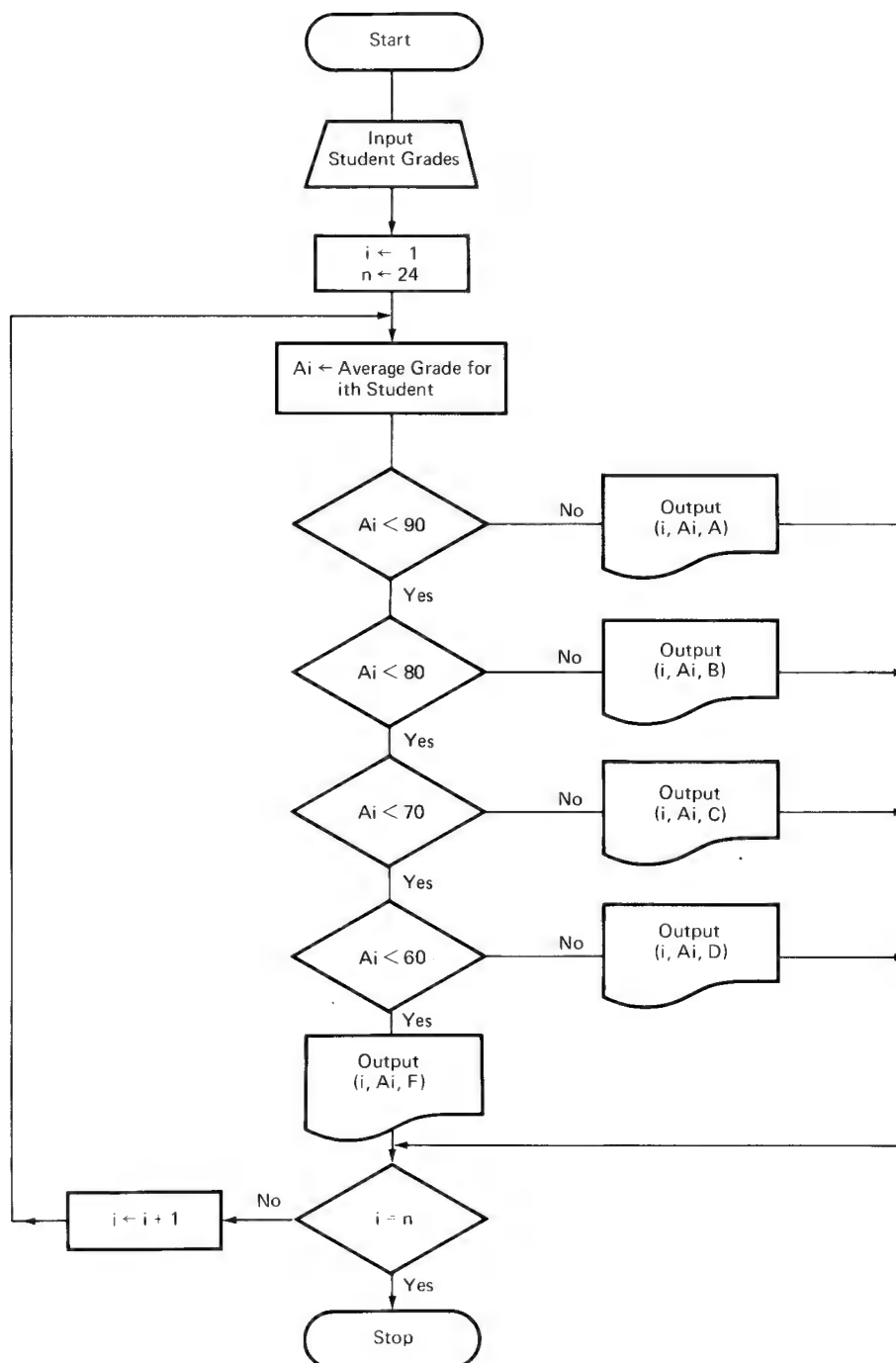
RUN

(STUDENT NO. 1	, 90.6667)
(STUDENT NO. 2	, 97.1667)
(STUDENT NO. 3	, 68.1667)
(STUDENT NO. 4	, 84)
(STUDENT NO. 5	, 86.3333)
(STUDENT NO. 6	, 77)
(STUDENT NO. 7	, 96.6667)
(STUDENT NO. 8	, 94.6667)
(STUDENT NO. 9	, 81.8333)
(STUDENT NO. 10	, 86)
(STUDENT NO. 11	, 94.6667)
(STUDENT NO. 12	, 78.3333)
(STUDENT NO. 13	, 80.8333)
(STUDENT NO. 14	, 86.6667)
(STUDENT NO. 15	, 64.1667)
(STUDENT NO. 16	, 64.3333)
(STUDENT NO. 17	, 68.5)
(STUDENT NO. 18	, 80.3333)
(STUDENT NO. 19	, 71.8333)
(STUDENT NO. 20	, 67.6667)
(STUDENT NO. 21	, 77.3333)
(STUDENT NO. 22	, 75.3333)
(STUDENT NO. 23	, 75.6667)
(STUDENT NO. 24	, 61.1667)

DONE

Micro Flow Chart

Exercise 1(b)



Example Program

Exercise 1(b)

LIST

```
10 REM--THIS PROGRAM WILL COMPUTE THE AVERAGE GRADE FOR A CLASS
20 REM--OF N STUDENTS BASED ON T GRADES FOR ONE GRADING PERIOD.
30 REM--THE OUTPUT IS IN ORDERED PAIR (FUNCTION) FORM. ENTER
40 REM --ON THE DATA LINE N, T, AND THE TEST SCORES OF EACH
50 REM--STUDENT, NO.1 THROUGH STUNDENT NO. N.
60 DATA 24,6,100,90,98,72,96,88,100,95,90,100,98,100,60,80,62,63,76,68
70 DATA 100,70,84,90,80,80,100,90,92,70,88,78,85,65,84,70,84,74
80 DATA 100,100,94,92,94,100,100,95,90,87,96,100,100,70,73,80,88,80
90 DATA 100,75,82,87,84,88,100,90,88,100,94,96,70,75,81,80,84,80
100 DATA 100,65,88,72,80,80,85,90,88,87,86,84,80,70,46,55,70,64,75,70,8
110 DATA 57,70,56,80,65,66,66,74,60,100,78,73,75,60,96,95,55,61,60,60
120 DATA 100,90,61,57,58,60,80,85,85,70,80,64,80,80,80,40,68,84
130 DATA 100,50,84,75,84,76,85,50,54,68,50,85,60,56,55,74,68
140 DIM A(200)
150 READ N,T
160 FOR I=1 TO N
170 LET G=0
180 FOR J=1 TO T
190 READ G[J]
200 LET G=G+G[J]
210 NEXT J
220 LET A[I]=G/T
230 IF A[I]<90 THEN 260
240 PRINT "(STUDENT NO.";I;",";"A[I];" A)"
250 GOTO 360
260 IF A[I]<80 THEN 290
270 PRINT "(STUDENT NO.";I;",";"A[I];" B)"
280 GOTO 360
290 IF A[I]<70 THEN 320
300 PRINT "(STUDENT NO.";I;",";"A[I];" C)"
310 GOTO 360
320 IF A[I]<60 THEN 350
330 PRINT "(STUDENT NO.";I;",";"A[I];" D)"
340 GOTO 360
350 PRINT "(STUDENT NO.";I;",";"A[I];" F)"
360 NEXT I
370 END
```

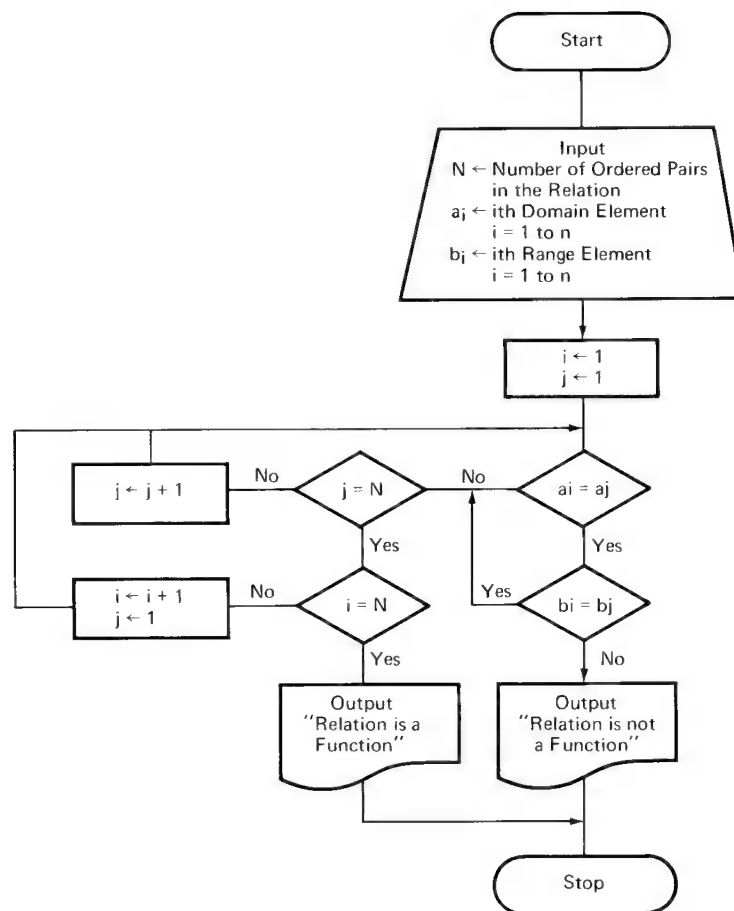
(STUDENT NO. 1	, 90.6667	A)
(STUDENT NO. 2	, 97.1667	A)
(STUDENT NO. 3	, 68.1667	D)
(STUDENT NO. 4	, 84 B)	
(STUDENT NO. 5	, 86.3333	B)
(STUDENT NO. 6	, 77 C)	
(STUDENT NO. 7	, 96.6667	A)
(STUDENT NO. 8	, 94.6667	A)
(STUDENT NO. 9	, 81.8333	B)
(STUDENT NO. 10	, 86 B)	
(STUDENT NO. 11	, 94.6667	A)
(STUDENT NO. 12	, 78.3333	C)
(STUDENT NO. 13	, 80.8333	B)
(STUDENT NO. 14	, 86.6667	B)
(STUDENT NO. 15	, 64.1667	D)
(STUDENT NO. 16	, 64.3333	D)
(STUDENT NO. 17	, 68.5	D)
(STUDENT NO. 18	, 80.3333	B)
(STUDENT NO. 19	, 71.8333	C)
(STUDENT NO. 20	, 67.6667	D)
(STUDENT NO. 21	, 77.3333	C)
(STUDENT NO. 22	, 75.3333	C)
(STUDENT NO. 23	, 75.6667	C)
(STUDENT NO. 24	, 61.1667	D)

DONE

Exercise 2 – Modeling the Definition of a Function

Micro Flow Chart

Exercise 2



Example Program

Exercise 2

```
1  DATA 6,3,2,-6,8,3,1,8,16,-.5,-2,4,2,7,7,1,-8,6,14,2,.8,3,6,3,10
2  DATA 0,-8,6,5,2,5,-3,.75,17,.3,2,-.7,3,5
10 REM--THIS PROGRAM DETERMINES IF A GIVEN FINITE SET OF ORDERED
11 REM--PAIRS IS A FUNCTION.  ENTER THE NUMBER OF ORDERED PAIRS
12 REM--N, IN THE RELATION, THEN THE ORDERED PAIRS PAIR BY PAIR.
30 READ N
40 FOR I=1 TO N
50 READ A[I],B[I]
60 NEXT I
100 FOR I=1 TO N
110 FOR J=1 TO N
120 IF A[I]=A[J] THEN 150
130 IF J=N THEN 180
140 NEXT J
150 IF B[I]=B[J] THEN 130
160 PRINT "RELATION IS NOT A FUNCTION"
170 GOTO 30
180 IF I=N THEN 200
190 NEXT I
200 PRINT "THE RELATIONS IS A FUNCTION"
205 GOTO 30
210 END
```

RUN

RELATION IS NOT A FUNCTION
THE RELATIONS IS A FUNCTION
RELATION IS NOT A FUNCTION

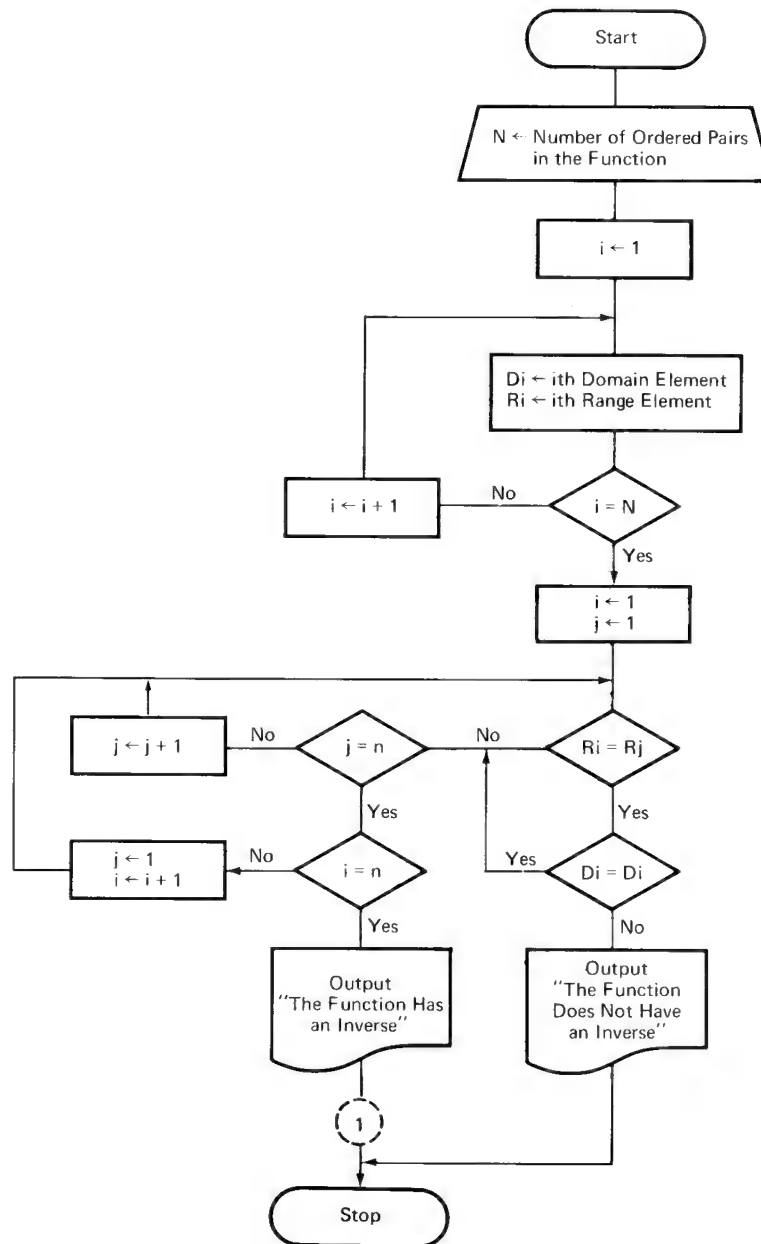
OUT OF DATA IN LINE 30

USING INFINITE FUNCTIONS TO INVESTIGATE FUNCTION PROPERTIES

Exercise 3 — Finding the Inverse of a Function

Micro Flow Chart

Exercise 3(a)



Example Program

Exercise 3(a)

```

10 DATA 5,-1,2,-3,5,7,-2,-3,5,.8,-.6
20 REM--THIS PROGRAM WILL DETERMINE IF A GIVEN FUNCTION DEFINED
30 REM--BY A FINITE SET OF ORDERED PAIRS HAS AN INVERSE. ENTER
40 REM--ON THE DATA LINE THE NUMBER OF ORDERED PAIRS IN THE SET
50 REM--FOLLOWED BY THE ORDERED PAIRS.
60 READ N
70 FOR I=1 TO N
80 READ D[I],R[I]
90 NEXT I
100 FOR I=1 TO N
110 FOR J=1 TO N
120 IF R[I]=R[J] THEN 160
130 NEXT J
140 NEXT I
150 GOTO 190
160 IF D[I]=D[J] THEN 130
170 PRINT "THE FUNCTION DOES NOT HAVE AN INVERSE"
180 GOTO 60
190 PRINT "THE FUNCTION HAS AN INVERSE"
200 GOTO 60
210 END

```

RUN

THE FUNCTION HAS AN INVERSE

OUT OF DATA IN LINE 60

10 DATA 6,7,15,6,2,-7,3,2,1,0,15,-.5,10

RUN

THE FUNCTION DOES NOT HAVE AN INVERSE

OUT OF DATA IN LINE 60

10 DATA 5,6,3,8,2,-1,6,-.8,.2,3,-7

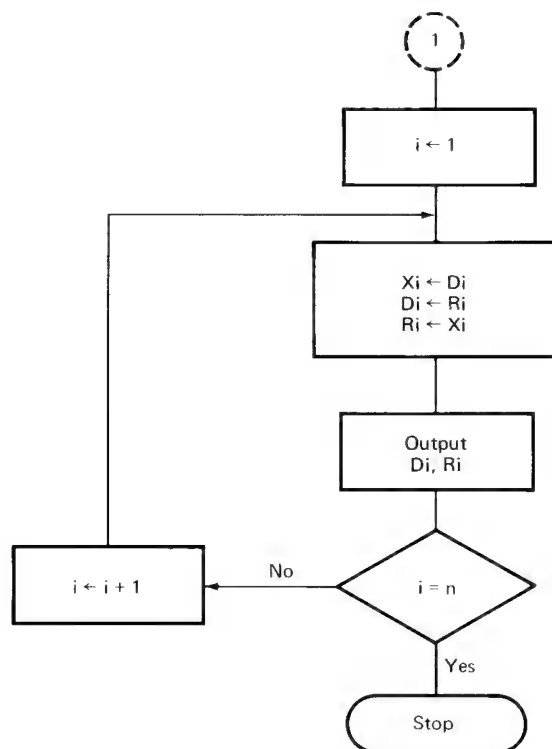
RUN

THE FUNCTION HAS AN INVERSE

OUT OF DATA IN LINE 60

(b) The student is asked to write a program that will cause an interchange of variable assignments of corresponding range elements. This is not necessary for an output of the inverse of a set of ordered pairs, but the exercise serves as an introduction to the interchange technique required in a later exercise.

Micro Flow Chart
Exercise 3(b)



Insert this section where indicated in
the flow chart of (a) above

Example Program

Exercise 3(b)

```
10 DATA 5,6,3,8,2,-1,6,-.8,.2,3,-7,6,7,15,6,2,-7,3,2,1,0,15,-.5,10
20 DATA 5,-1,2,-3,5,7,-2,-3,5,.8,-.6
30 REM--THIS PROGRAM WILL PRINT THE INVERSE OF A GIVEN
40 REM--FUNCTION, IF IT EXISTS.
50 REM-- ENTER ON THE DATA LINE THE NUMBER OF
60 REM--ORDERED PAIRS IN THE SET, THE ORDERED PAIRS, PAIR
70 REM--BY PAIR.
80 READ N
90 FOR I=1 TO N
100 READ D[I],R[I]
110 NEXT I
120 FOR I=1 TO N
130 FOR J=1 TO N
140 IF R[I]=R[J] THEN 180
150 NEXT J
160 NEXT I
170 GOTO 220
180 IF D[I]=D[J] THEN 150
190 PRINT "THE FUNCTION DOES NOT HAVE AN INVERSE"
200 PRINT ""
210 GOTO 80
220 PRINT "THE FUNCTION HAS AN INVERSE"
230 PRINT ""
240 FOR I=1 TO N
250 LET X[I]=D[I]
260 LET D[I]=R[I]
270 LET R[I]=X[I]
280 PRINT "(";D[I];",";R[I];")"
290 NEXT I
300 PRINT ""
310 GOTO 80
320 END
```

RUN

THE FUNCTION HAS AN INVERSE

```
( 3      , 6      )
( 2      , 8      )
( 6      , -1     )
( .2     , -.8     )
(-7     , 3      )
```

THE FUNCTION DOES NOT HAVE AN INVERSE

THE FUNCTION HAS AN INVERSE

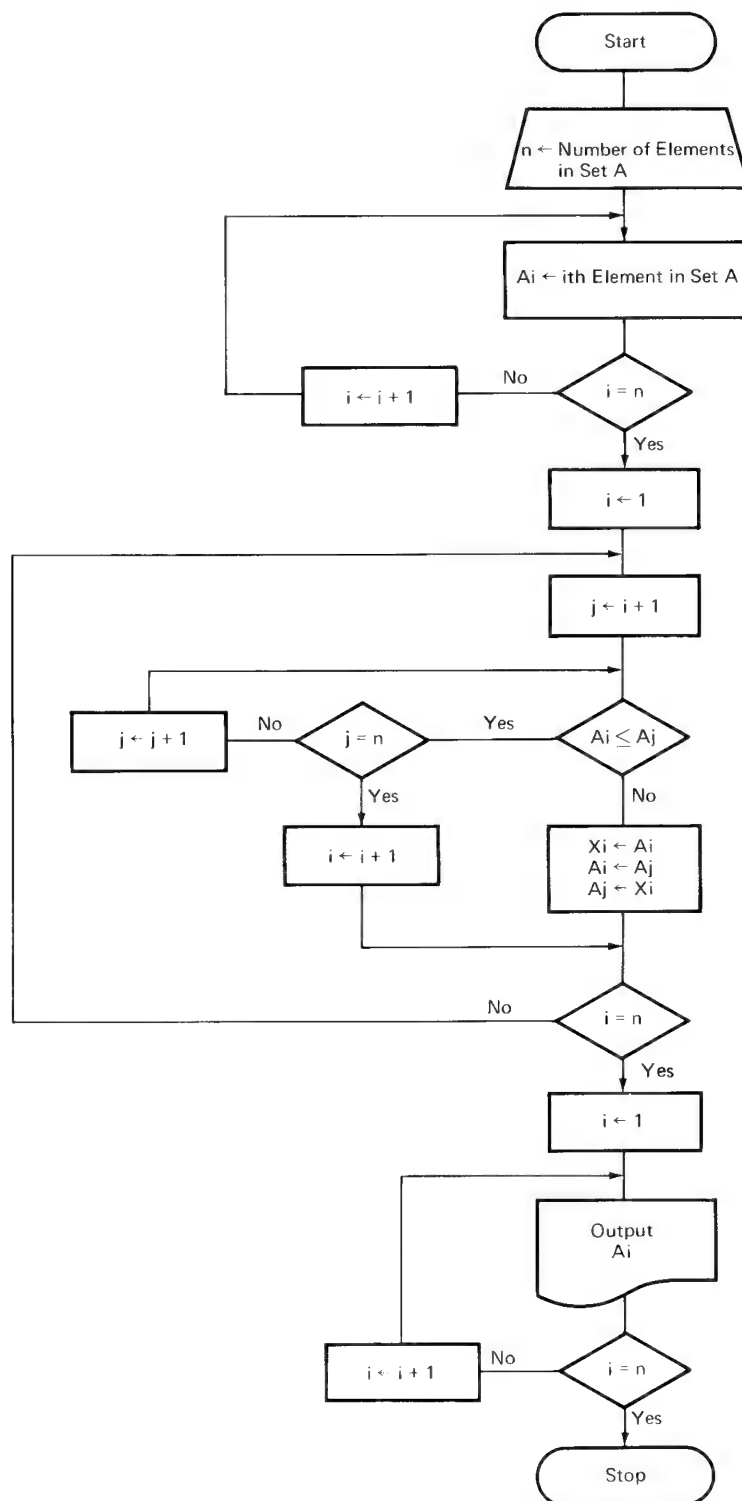
```
( 2      , -1     )
( 5      , -3     )
(-2     , 7      )
( 5      , -3     )
(-.6    , .8      )
```

OUT OF DATA IN LINE 80

Exercise 4 — Arranging Numbers in Ascending Order

Micro Flow Chart

Exercise 4



Example Program

Exercise 4

```
1  DATA 6,6,-2,8,5,0,-8
10 REM--THIS PROGRAM WILL ARRANGE A FINITE SET OF REAL NUMBERS
11 REM--IN ASCENDING ORDER.  ON THE DATA LINE ENTER THE NUMBER
12 REM--OF ELEMENTS IN THE SET AND THE ELEMENTS OF THE SET.
20 READ N
30 FOR I=1 TO N
40 READ A[I]
50 NEXT I
60 I=1
70 J=I+1
80 IF A[I] <= A[J] THEN 140
90 X[I]=A[I]
100 A[I]=A[J]
110 A[J]=X[I]
120 IF I=N THEN 170
130 GOTO 70
140 IF J=N THEN 210
150 J=J+1
160 GOTO 80
170 FOR I=1 TO N
180 PRINT A[I]
190 NEXT I
200 GOTO 230
210 I=I+1
220 GOTO 120
230 END
```

RUN

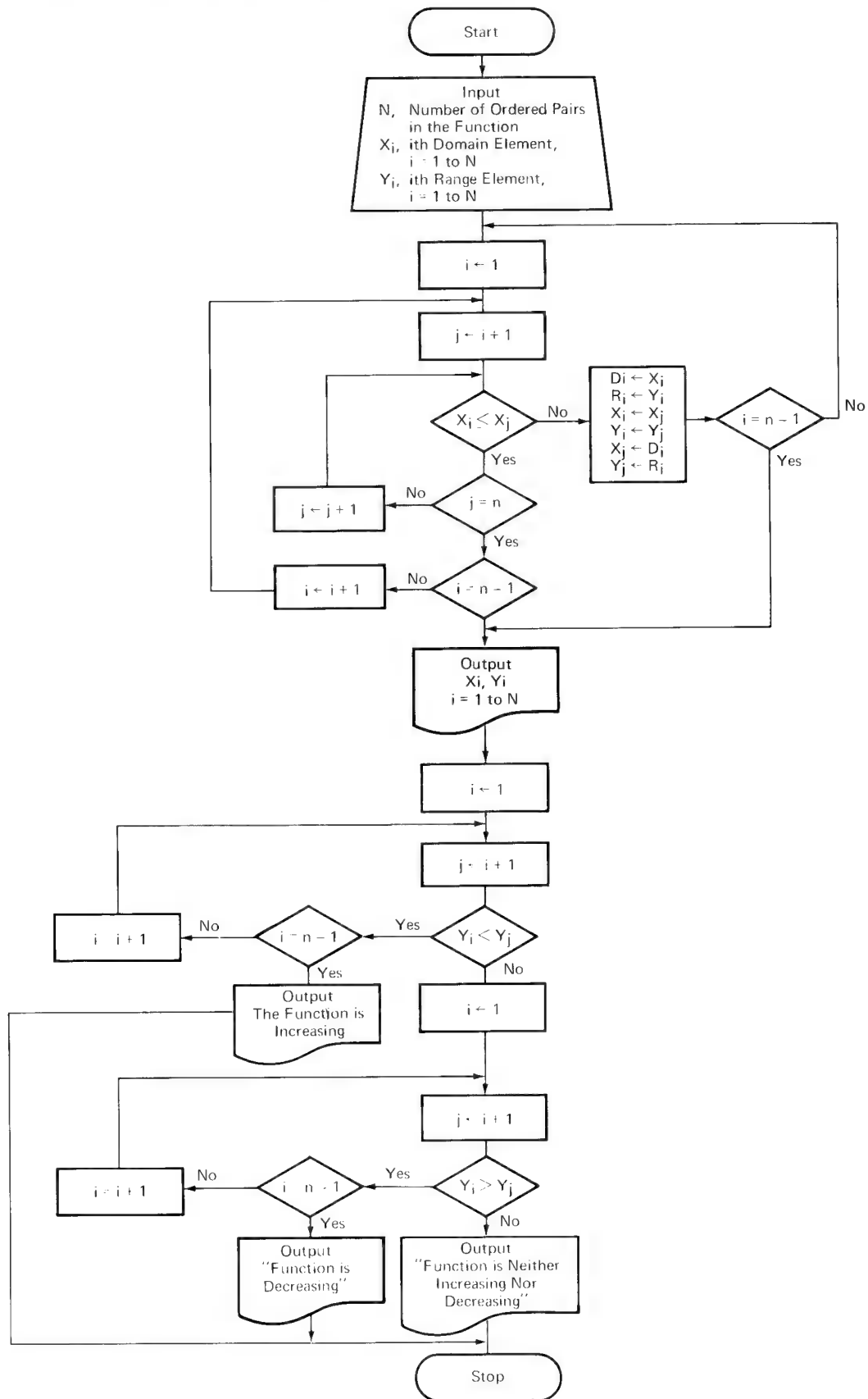
```
-8
-2
0
5
6
8
```

DONE

Exercise 5 — Identifying Increasing and Decreasing Functions

Micro Flow Chart

Exercise 5(a)



Example Program

Exercise 5(a)

```
10 DATA 7,.5,0,1.75,2,-1,-7,0,-4,-.5,-7,-3,-8,3,15
20 DATA 6,.5,-1,-1,0,7,-22,-7,6,3.5,-6,-5,0
30 DATA 6,-6,7,-5,2,-4,0,-3,2,0,0,3,1
40 REM--A PROGRAM TO DETERMINE IF A FUNCTION IS "INCREASING",
50 REM--"DECREASING" OR NEITHER. ON THE DATA LINE ENTER THE
60 REM--NUMBER OF ORDERED PAIRS IN THE SET FOLLOWED BY THE
65 REM--ORDERED PAIRS OF THE SET.
70 READ N
80 FOR I=1 TO N
90 READ X[I],Y[I]
100 NEXT I
110 LET I=1
120 LET J=I+1
130 IF X[I] <= X[J] THEN 220
140 LET D[I]=X[I]
150 LET R[I]=Y[I]
160 LET X[I]=X[J]
170 LET Y[I]=Y[J]
180 LET X[J]=D[I]
190 LET Y[J]=R[I]
200 IF I=N-1 THEN 250
210 GOTO 110
220 IF J=N THEN 370
230 LET J=J+1
240 GOTO 130
250 FOR I=1 TO N
260 PRINT X[I],Y[I]
270 NEXT I
280 I=1
290 J=I+1
300 IF Y[I] <= Y[J] THEN 400
310 I=1
320 J=I+1
330 IF Y[I] >= Y[J] THEN 430
340 PRINT "FUNCTION IS NEITHER INCREASING OR DECREASING"
350 PRINT
360 GOTO 70
370 IF I=N-1 THEN 250
380 I=I+1
390 GOTO 120
400 IF I=N-1 THEN 460
410 I=I+1
420 GOTO 290
430 IF I=N-1 THEN 490
440 I=I+1
450 GOTO 320
460 PRINT "FUNCTION IS INCREASING"
470 PRINT
480 GOTO 70
490 PRINT "FUNCTION IS DECREASING"
500 PRINT
510 GOTO 70
520 END
```


RUN

-3	-8
-1	-7
-.5	-7
0	-4
.5	0
1.75	2
3	15

FUNCTION IS INCREASING

-7	6
-5	0
-1	0
.5	-1
3.5	-6
7	-22

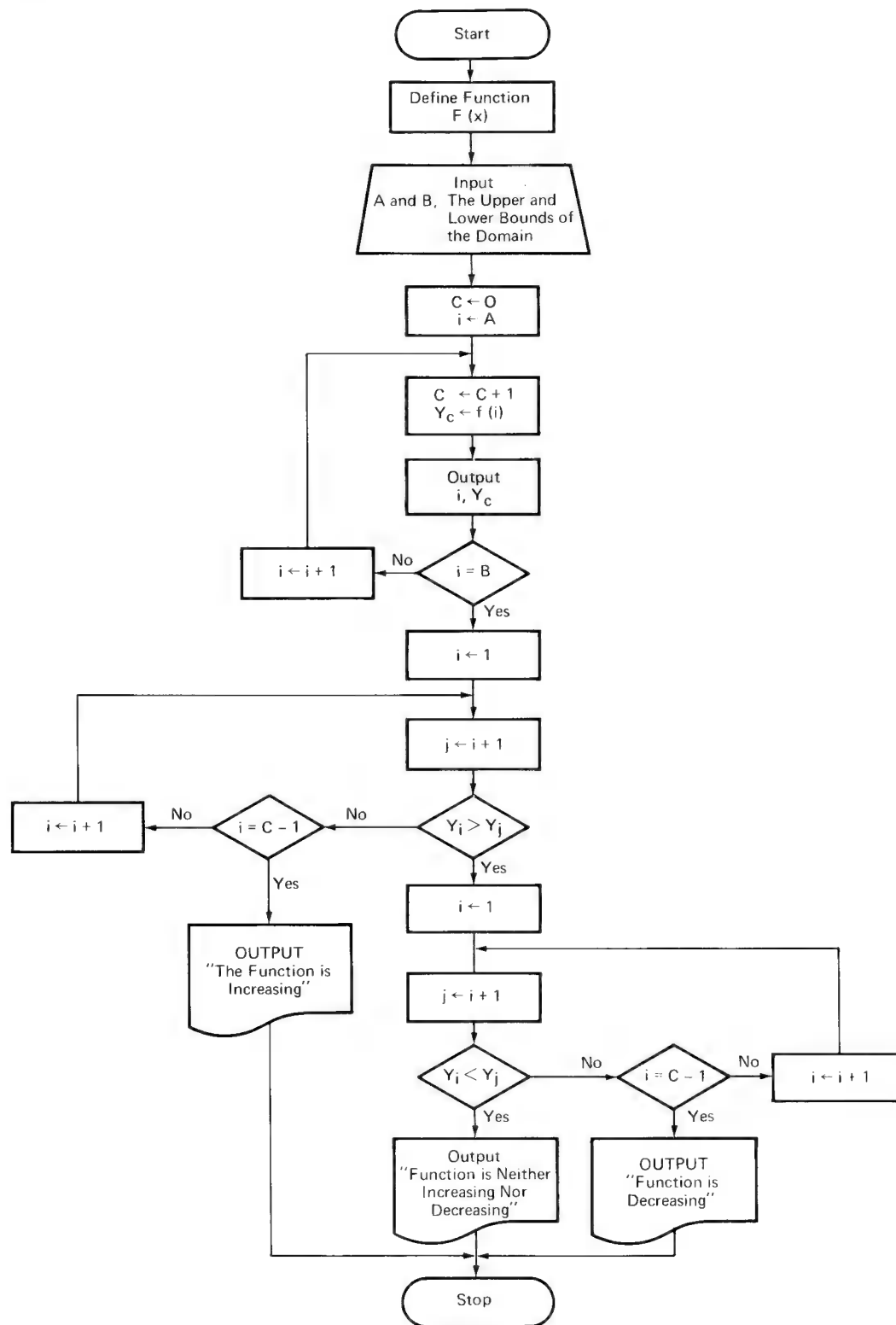
FUNCTION IS DECREASING

-6	7
-5	2
-4	0
-3	2
0	0
3	1

FUNCTION IS NEITHER INCREASING OR DECREASING

OUT OF DATA IN LINE 70

Micro Flow Chart
Exercise 5(b)



Example Program

Exercise 5(b)

```
10 REM--A PROGRAM TO DETERMINE IF A FUNCTION IS "INCREASING",
20 REM--"DECREASING", OR NEITHER. ON LINE 60
30 REM--DEFINE THE FUNCTION. INPUT UPPER AND LOWER DOMAIN.
35 C=0
40 DIM X[20],Y[20]
50 INPUT A,B
60 DEF FNP(X)=X^2-X
70 FOR I=A TO B
80 C=C+1
90 Y[C]=FNP(I)
95 PRINT I;Y[C]
100 NEXT I
110 FOR I=1 TO C-1
120 J=I+1
130 IF Y[I]>Y[J] THEN 180
140 NEXT I
150 PRINT "THE FUNCTION IS INCREASING"
160 PRINT
170 GOTO 270
180 FOR I=1 TO C-1
190 J=I+1
200 IF Y[I]<Y[J] THEN 250
210 NEXT I
220 PRINT "THE FUNCTION IS DECREASING"
230 PRINT
240 GOTO 270
250 PRINT "THE FUNCTION IS NEITHER INCREASING OF DECREASING"
260 PRINT
270 END
```

RUN

?-3,3

-3	12
-2	6
-1	2
0	0
1	0
2	2
3	6

THE FUNCTION IS NEITHER INCREASING OF DECREASING

DONE

RUN

?0,5

0	0
1	0
2	2
3	6
4	12
5	20

THE FUNCTION IS INCREASING

DONE

60 DEF FNP(X)=6-X*2

RUN

?-5,5

-5	16
-4	14
-3	12
-2	10
-1	8
0	6
1	4
2	2
3	0
4	-2
5	-4

THE FUNCTION IS DECREASING

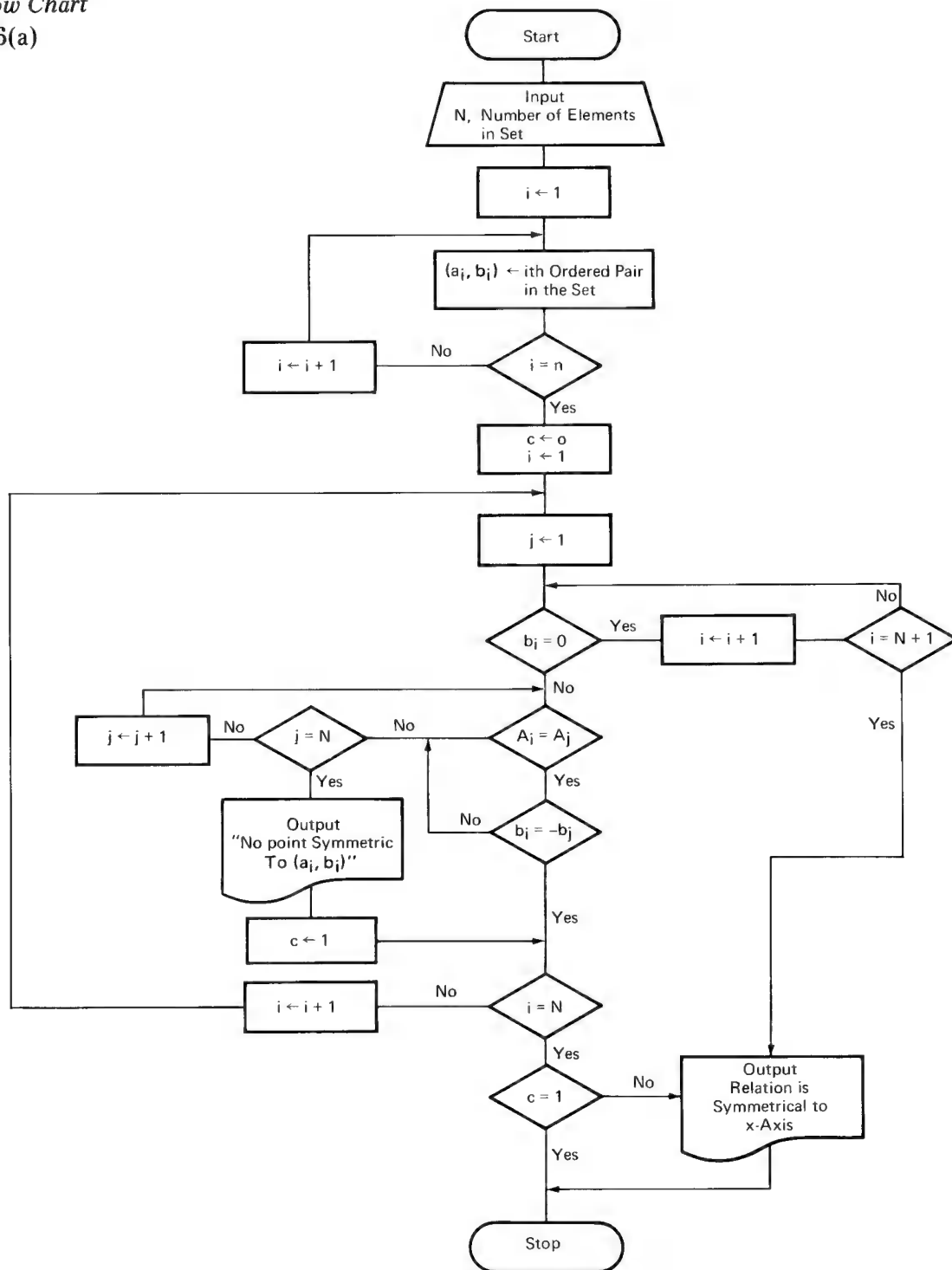
DONE

Exercise 6 — Symmetry

In this program, make sure that the students provide for the points $(0,b)$, $(a,0)$ $(0,0)$ to map into themselves for x-axis, y-axis, and origin symmetry, respectively.

Micro Flow Chart

Exercise 6(a)



Example Program

Exercise 6(a)

```

10 REM--A PROGRAM TO MODEL THE DEFINITION OF SYMMETRY ABOUT
20 REM--THE X-AXIS. ENTER ON THE DATA LINE N, THE NUMBER OF
30 REM--ORDERED PAIRS IN THE SET AND THE ELEMENTS OF THE
40 REM--ORDERED PAIRS, PAIR BY PAIR.
50 DATA 9,6,3,-2,-1,-2,3,.7,-5,-2,1,.7,5,6,-3,-3,-2,-2,-3
60 DATA 7,2,15,7,0,6,-1,.5,-2,6,1,2,-15,.5,2
70 DATA 9,-5,1,7,3,6,.3,0,0,0,-2,6,-.3,7,-3,-5,-1,0,2
80 READ N
90 FOR I=1 TO N
100 READ A[I],B[I]
110 NEXT I
120 LET C=0
130 LET I=1
140 LET J=1
150 IF B[I]=0 THEN 200
160 IF A[I]=A[J] THEN 230
170 IF J=N THEN 250
180 LET J=J+1
190 GOTO 160
200 LET I=I+1
210 IF I=N+1 THEN 330
220 GOTO 150
230 IF B[I]=-B[J] THEN 290
240 GOTO 170
250 PRINT "NO POINT SYMMETRIC TO (";A[I];",";B[I];")"
270 LET C=1
290 IF I=N THEN 320
300 LET I=I+1
310 GOTO 140
320 IF C=1 THEN 340
330 PRINT "RELATION IS SYMMETRICAL ABOUT THE X-AXIS"
340 GOTO 80
350 END

```

RUN

```

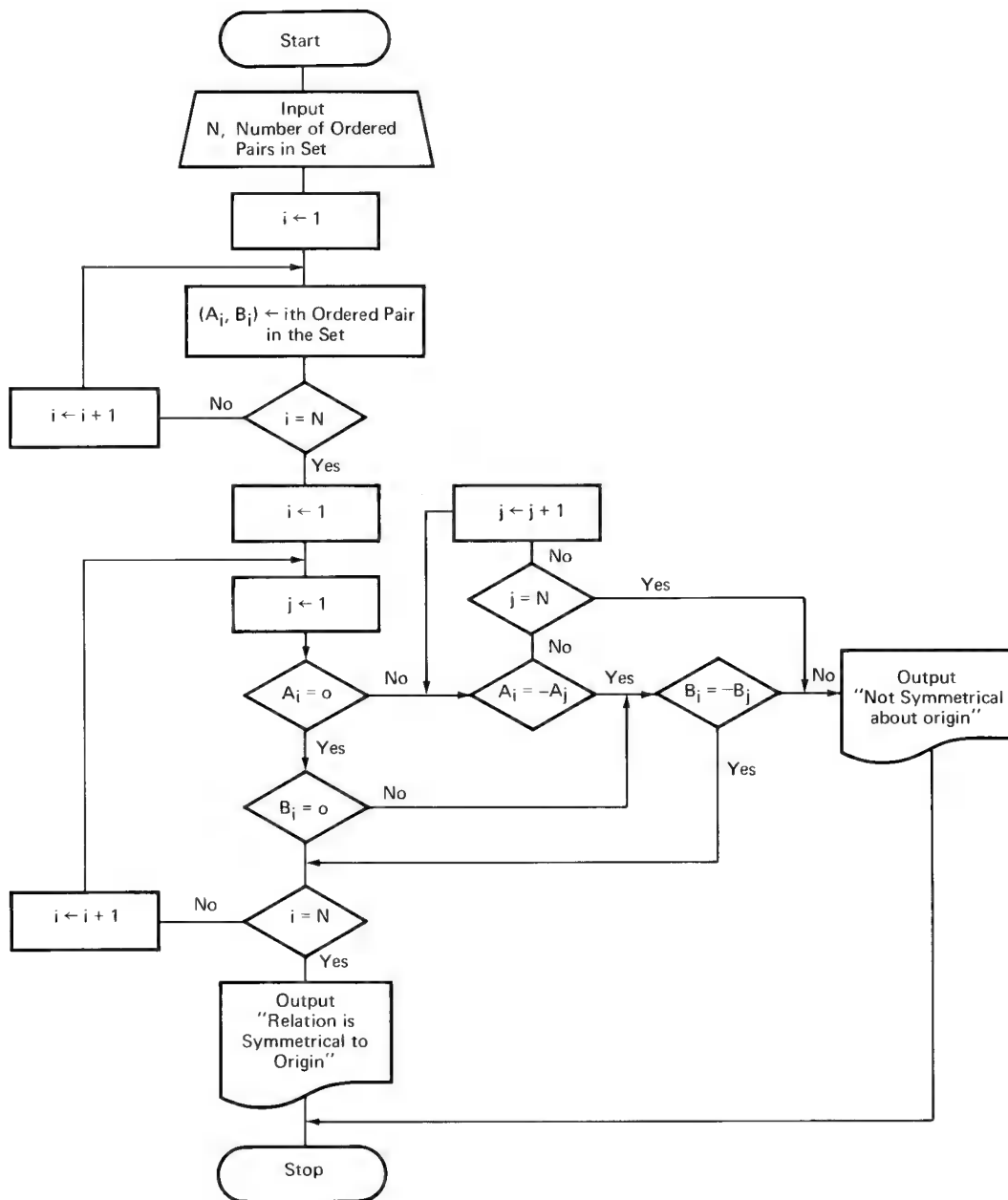
NO POINT SYMMETRIC TO (-3      ,-2      )
RELATION IS SYMMETRICAL ABOUT THE X-AXIS
RELATION IS SYMMETRICAL ABOUT THE X-AXIS

```

OUT OF DATA IN LINE 80

Micro Flow Chart

Exercise 6(b)



Example Program

Exercise 6(b)

```

10 REM--THIS PROGRAM MODELS THE DEFINITION OF SYMMETRY ABOUT
11 REM--THE ORIGIN.  ENTER, ON THE DATA LINE N, THE NUMBER OF
12 REM --THE ORDERED PAIRS AND THE ORDERED PAIRS, PAIR BY PAIR.
20 DATA 7,-7,1,-6,2,-5,3,-4,4,5,-3,6,-2,7,-1
30 READ N
40 FOR I=1 TO N
50 READ A[I],B[I]
60 NEXT I
70 LET I=1
80 LET J=1
90 IF A[I]=0 THEN 140
100 IF A[I]=-A[J] THEN 150
110 IF J=N THEN 160
120 LET J=J+1
130 GOTO 100
140 IF B[I]=0 THEN 200
150 IF B[I]=-B[J] THEN 200
160 PRINT "RELATION NOT SYMMETRICAL ABOUT THE ORIGIN"
163 GOTO 250
200 IF I=N THEN 240
210 LET I=I+1
220 GOTO 80
240 PRINT "RELATION IS SYMMETRICAL ABOUT ORIGIN"
250 END

```

RUN

RELATION NOT SYMMETRICAL ABOUT THE ORIGIN

DONE

20 DATA 8,-7,1,-6,2,-5,3,-4,4,7,-1,6,2,5,-3,4,-4

RUN

RELATION NOT SYMMETRICAL ABOUT THE ORIGIN

DONE

20 DATA 9,-7,1,-6,2,-5,3,-4,4,7,-1,6,-2,5,-3,4,-4,0,0

RUN

RELATION IS SYMMETRICAL ABOUT ORIGIN

DONE

20 DATA 9,-7,1,-6,2,-5,3,0,4,3,1,7,-1,6,-2,5,-3,-3,-1

RUN

RELATION NOT SYMMETRICAL ABOUT THE ORIGIN

DONE

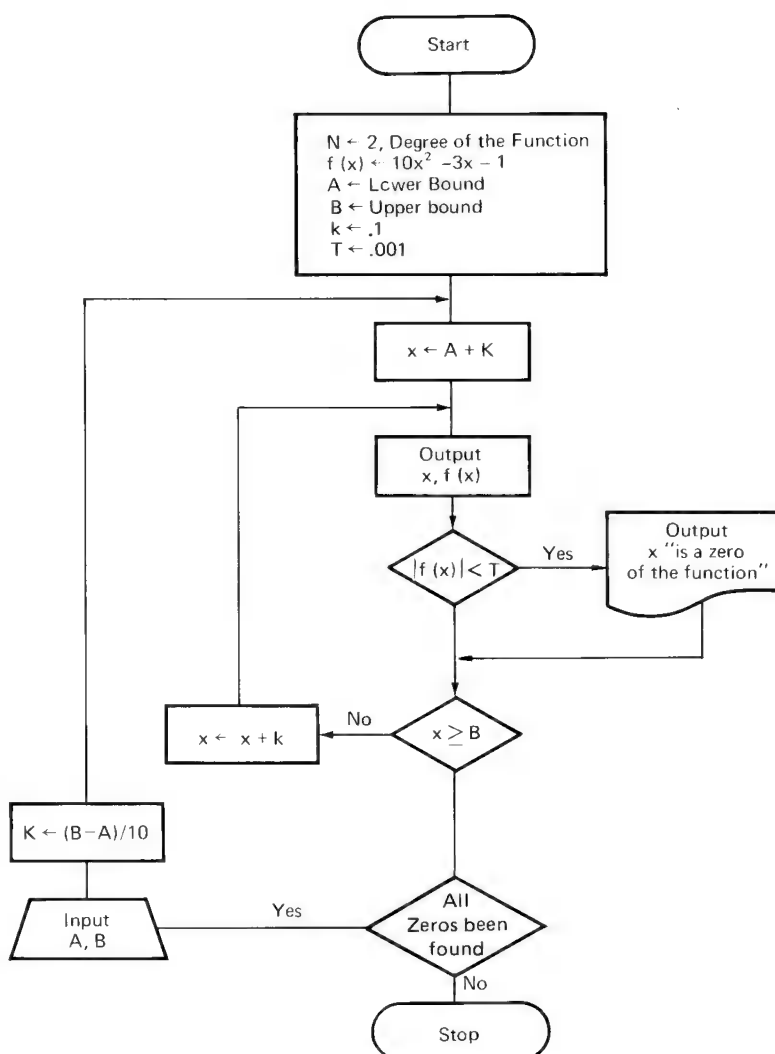
USING INFINITE FUNCTIONS TO INVESTIGATE OTHER FUNCTION PROPERTIES

Exercise 7 — The Zeros of a Function

This exercise is not intended to present a reliable method for finding zeros of all polynomials. Its main purpose is to convey the concept of a zero. The unreliability of this method does provide opportunity for discussion of computer error and problems associated with “not well-behaved” curves.

The following program provides a method to input new upper and lower bounds of an interval if a search for a given increment does not determine all zeros.

Micro Flow Chart Exercise 7



Example Program

Exercise 7

```
10 REM--A PROGRAM TO DETERMINE THE ZEROS OF A POLYNOMIAL.
20 REM--ON LINE 40 ENTER THE POLYNOMIAL.
40 DEF FNF(X)=X3+3+3*X-1
50 PRINT "ENTER LOWER BOUND AND UPPER BOUND OF INTERVAL BEING SEARCHED."
60 PRINT "ALSO, THE TOLERANCE DESIRED FOR APPROXIMATE ZEROS."
70 INPUT A,B,T
80 K=.1
90 X=A+K
100 PRINT X,FNF(X)
110 IF ABS(FNF(X))<T THEN 150
120 IF X >= B THEN 190
130 X=X+K
140 GOTO 100
150 PRINT ""
160 PRINT X;"IS A ZERO"
170 PRINT ""
180 GOTO 120
190 PRINT "IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'"
200 PRINT "EXAMINE THE ABOVE OUTPUT,ENTER THE LOWER AND UPPER BOUND"
210 PRINT "OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION"
220 PRINT "VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'."
240 INPUT Y
250 IF Y=1 THEN 290
260 INPUT A,B
270 K=(B-A)/10
280 GOTO 90
290 END
```

```
40 DEF FNF(X)=10*X^2-3*X-1
RUN
```

ENTER LOWER BOUND AND UPPER BOUND OF INTERVAL BEING SEARCHED.
ALSO, THE TOLERANCE DESIRED FOR APPROXIMATE ZEROS.

```
?-1,1,.001
```

-.9	9.8
-.8	7.8
-.7	6.
-.6	4.4
-.5	3.
-.4	1.8
-.3	.8
-.2	2.38419E-07

```
-.2          IS A ZERO
```

-.1	-.6
-2.98023E-08	-1.
.1	-1.2
.2	-1.2
.3	-1.
.4	-.6
.5	0

```
.5          IS A ZERO
```

.6	.8
.7	1.8
.8	3.
.9	4.4
1	6

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'
EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.
?1

```
DONE
```

```
40 DEF FNF(X)=X+3+3*X-1
```

```
RUN
```

ENTER LOWER BOUND AND UPPER BOUND OF INTERVAL BEING SEARCHED.
ALSO, THE TOLERANCE DESIRED FOR APPROXIMATE ZEROS.

```
?0,1,.001
```

.1	-.699
.2	-.392
.3	-.073
.4	.264
.5	.625
.6	1.016
.7	1.443
.8	1.912
.9	2.429
1	3

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'

EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.

```
?0
```

```
? .3, .4
```

.31	-4.02089E-02
.32	-7.23195E-03
.33	2.59371E-02
.34	.059304
.35	.092875
.36	.126656
.37	.160653
.38	.194872
.39	.229319
.4	.264
.41	.298921

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'

EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.

```
?0
```

```
? .32, .33
```

.321	-3.92389E-03
.322	-6.13809E-04

```
.322      IS A ZERO
```

.323	2.69818E-03
.324	6.01220E-03
.325	9.32789E-03
.326	1.26457E-02
.327	1.59655E-02
.328	1.92873E-02
.329	2.26109E-02
.33	2.59366E-02
.331	2.92642E-02

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'
 EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
 OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
 VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.
 ?1

DONE

40 DEF FNF(X)=12*X^2-7*X+1

RUN

ENTER LOWER BOUND AND UPPER BOUND OF INTERVAL BEING SEARCHED.
 ALSO, THE TOLERANCE DESIRED FOR APPROXIMATE ZEROS.

?-1,1,.001

-.9	17.02
-.8	14.28
-.7	11.78
-.6	9.52
-.5	7.5
-.4	5.72
-.3	4.18
-.2	2.88
-.1	1.82
-2.98023E-08	1.
.1	.42
.2	8.00002E-02
.3	-2.00002E-02
.4	.12
.5	.5
.6	1.12
.7	1.98
.8	3.08
.9	4.42
1	6

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'
 EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
 OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
 VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.

?0

? .2, .3

.21	5.92002E-02
.22	4.08001E-02
.23	2.48001E-02
.24	1.12002E-02
.25	1.19209E-07

.25 IS A ZERO

.26	-8.79979E-03
.27	-1.51999E-02
.28	-1.91998E-02
.29	-2.07999E-02
.3	-2.00002E-02
.31	-1.67999E-02

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'
 EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
 OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
 VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.

?0

? .3, .4

.31	-1.67999E-02
.32	-.0112
.33	-3.20005E-03
.34	.0072
.35	.02
.36	3.52001E-02
.37	5.27997E-02
.38	7.27999E-02
.39	9.51998E-02
.4	.12
.41	.1472

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'
 EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
 OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
 VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.

?0

? .33, .34

.331	-2.26808E-03
.332	-1.31202E-03
.333	-3.32355E-04

.333 IS A ZERO

.334 6.72102E-04

.334 IS A ZERO

.335 1.70016E-03

.336 2.75207E-03

.337 3.82805E-03

.338 4.92787E-03

.339 6.05178E-03

.34 7.19976E-03

.341 8.37159E-03

IF ALL ZEROS HAVE NOT BEEN FOUND, TYPE '0'

EXAMINE THE ABOVE OUTPUT, ENTER THE LOWER AND UPPER BOUND
OF THE INTERVAL IN WHICH A SIGN CHANGE IN THE FUNCTION
VALUES OCCURS. IF ALL ZEROS HAVE BEEN FOUND, TYPE '1'.

?1

DONE

Exercise 8 — Discontinuous Functions

(a), (b), (c). Since this exercise really requires only finding the zeros of a polynomial function, we will not write a separate program for this problem. The program written for Exercise 7 can be used with the following change:

line 160 PRINT "THE FUNCTION IS DISCONTINUOUS AT X = "; X

(d) The equations $f(x) = \frac{x-1}{(x-1)(x+3)}$ and $g(x) = \frac{1}{x+3}$ do not define the same function because $f(x)$ is not defined at $x = 1$ while $g(x)$ is.

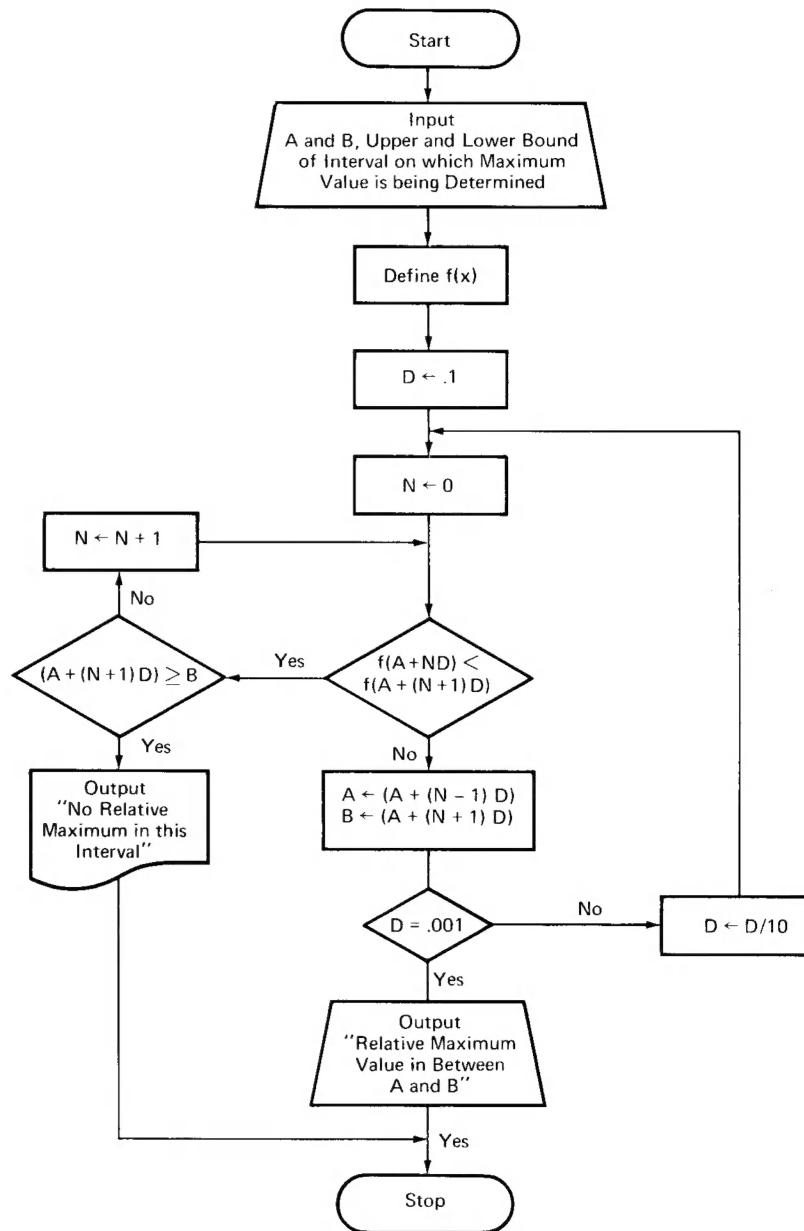
Exercise 9 — Relative Maximum Value

(a) You will notice that the technique used to approximate a relative maximum on an interval is not adequate unless the curve is "well-behaved" on that interval. Nevertheless, it is presented here with a well-behaved example since it quite effectively illustrates the concept of a relative maximum.

(b) This problem illustrates that relative maximum values do not exist for all intervals on a function. Students probably did not provide for this condition in the program for part (a), so they will need to modify their programs. The example program listed for part (a) does provide for intervals on which no relative maximums exist.

(c) This practical application of the program of part (a) should be of interest to the students.

Micro Flow Chart
Exercise 9(a)



Example Program

Exercise 9(a)

```
10 REM--A PROGRAM TO DETERMINE THE RELATIVE MAXIMUM VALUE OF
11 REM--A POLYNOMIAL FUNCTION F. ON THE DATA LINE ENTER THE
12 REM--INTERVAL BOUNDS A AND B ON WHICH THE MAXIMUM VALUE IS
13 REM--TO BE DETERMINED.
20 DEF FNP(X)=X^3-2*X^2-X+2
30 READ A,B
40 DATA -1,1
50 LET D=.1
60 LET N=0
70 IF FNP(A+N*D)<FNP(A+(N+1)*D) THEN 130
80 LET A=A+(N-1)*D
90 LET B=A+(N+1)*D
100 IF D=.001 THEN 160
110 LET D=D/10
120 GOTO 60
130 IF (A+(N+1)*D) >= B THEN 180
140 LET N=N+1
150 GOTO 70
160 PRINT "RELATIVE MAXIMUM IS APPROXIMATELY ";FNP(A+N*D)
170 GOTO 190
180 PRINT "NO RELATIVE MAXIMUM ON THE GIVEN INTERVAL"
190 END
```

RUN

RELATIVE MAXIMUM IS APPROXIMATELY 2.11208

DONE

```
20 DEF FNP(X)=-4*X^2+7*X+2
40 DATA 0,2
RUN
```

RELATIVE MAXIMUM IS APPROXIMATELY 5.06172

DONE

Example Program

Exercise 9(c)

```
10 REM--A PROGRAM TO DETERMINE THE RELATIVE MAXIMUM VALUE OF
11 REM--A POLYNOMIAL FUNCTION F. ON THE DATA LINE ENTER THE
12 REM--INTERVAL BOUNDS A AND B ON WHICH THE MAXIMUM VALUE IS
13 REM--TO BE DETERMINED.
20 DEF FNP(X)=4*X^3-42*X^2+108*X
30 READ A,B
40 DATA 1,2
50 LET D=.1
60 LET N=0
70 IF FNP(A+N*D)<FNP(A+(N+1)*D) THEN 130
80 LET A=A+(N-1)*D
90 LET B=A+(N+1)*D
100 IF D=.001 THEN 160
110 LET D=D/10
120 GOTO 60
130 IF (A+(N+1)*D) >= B THEN 180
140 LET N=N+1
150 GOTO 70
160 PRINT "THE MAXIMUM VOLUME IS APPROXIMATELY";FNP(A+N*D);
165 PRINT "WHEN THE DEMINSION OF THE SQUARES CUT OUT IS BETWEEN";
166 PRINT A;"AND";B;"INCHES"
170 GOTO 190
180 PRINT "NO RELATIVE MAXIMUM ON THE GIVEN INTERVAL"
190 END
```

RUN

THE MAXIMUM VOLUME IS APPROXIMATELY 81.8714
WHEN THE DEMINSION OF THE SQUARES CUT OUT IS BETWEEN 1.696 AND
1.704 INCHES

DONE

